(In press)
Journal for Research in Mathematics Education

Professionals Read Graphs: A Semiotic Analysis

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Abstract

Graph-related practices are central to scientific endeavors and graphing has long been hailed as one of the core “general process skills” that set scientists apart. One research question that has not received much attention is, “Are scientists generally competent readers of graphs, or are graphs indissolubly tied to practices and understandings of their everyday workplace?” This study was designed to better understand the reading and interpretation of familiar and unfamiliar graphs by (mostly) scientists. From an extensive database on graphing involving university students to professionals, we selected two case studies and interpret them within a theoretical framework grounded in semiotics and hermeneutic phenomenology. The first case study provides a detailed analysis in which a scientist wrestles and in part inappropriately interprets an unfamiliar graph used in undergraduate ecology courses; the difficulties of the scientist include some of those identified among students in the graphing literature. The second case study provides an example of the transparent use of graphs in the work of a water technician who is not only familiar with her graphs, but who has an intimate, embodied knowledge of the world to which the graph refers. When it comes to reasoning, scientists are often taken as experts against which the performance of other individuals (“novices”) are judged (as inferior). If scientists’ own graphing practices are not general but tied to their embodied understanding, then the teaching of decontextualized graphing skills loses its legitimacy. Our results therefore have considerable implications to mathematics education.

Introduction

Cartesian graphs are central to the representation of the world in the natural sciences. A recent analysis showed that there were more than 420 Cartesian graphs in 2,500 pages of five top-ranked ecology research journals (Roth, Bowen, & McGinn, 1999). Much of the ethnographic work in scientific laboratories conducted over the past two decades suggests that not only are graphs used to construct phenomena, but they also serve as an existence proof of the phenomena and, as thus, are employed as rhetorical means in scientific publications (Latour, 1987). Graphs are central to the constructive effort of scientists in establishing just what is seen in and evidenced by the unfolding pattern of a graph (Garfinkel, Lynch, & Livingston, 1981; Latour, 1993; Woolgar, 1990). Most importantly, in the natural sciences, graphs (as other mathematical representations) are used in a transparent way such that an isomorphism of the type “fundamental structure <—> mathematical structure” is presupposed (Lynch, 1991).

It therefore comes as no surprise that graph-related activities take an important place in recent reform efforts in mathematics and science education. Relative to the interpretation of existing graphs for the mathematics classroom in Grades 5 - 8, the Curriculum and Evaluation Standards (NCTM, 1989) state that students should be able to:

- describe and represent relationships with tables, graphs, and rules; (p. 98)
- construct, read, and interpret tables, charts, and graphs; (p. 105)
- analyze tables and graphs to identify properties and relationships. (p. 102)

In the past, most research took a cognitive psychological perspective which identified problems in students’ reading and interpreting graphs, and attributed these problems to “misconceptions” (e.g., Leinhardt, Zaslavsky, & Stein, 1990), “cognitive deficits” (e.g., Berg & Philips, 1994), and other more generic “deficiencies” (e.g., Preece & Janvier, 1992). More recent theoretical work on graphing, however, suggests that a sociocultural orientation toward graphing as practice avoids the deficit approach to students’ graphing-related actions (Roth, 1996; Roth & McGinn, 1997, 1998). This approach has become even more convincing and necessary as a recent study among scientists shows. When scientists are not familiar with a graph, even if this graph is from introductory textbooks of their own discipline (ecology), they frequently do not arrive at the collectively-accepted standard interpretation (Roth, 1998). Some of the breakdowns they encounter while reading graphs (and captions) are of the same type that have been identified among middle and high school students. However, because of their extensive training (M.Sc., Ph.D.), experience (minimum of 5 years independent research), and career-related success (many have received national and international fellowships, grants, and awards), it would be difficult to accept that these scientists hold “misconceptions,” have “cognitive deficits,” or suffer from other “deficiencies.” Here, to understand these problems, we take the notion of graphing as practice further. By fusing semiotics and hermeneutic phenomenology, we account both for individual aspects of reading graphs and for the social matrix within which each individual operates. That is, our framework allows us to link mathematical (graph-related) experience with experience in the world.

The purpose of the present article is to exemplify two ways in which scientists engaged with graphs; we present these types of engagement by drawing on two detailed case studies: one features a scientist who struggles with the interpretation of an unfamiliar graph (produced for us in the somewhat artificial situation associated with such research); the other features a water technician who reads familiar graphs in a transparent way as part of her ongoing work which included raising public awareness for water issues. Based on the case studies and the underlying theoretical framework, we consider the implications for changing graphing-related activities in school mathematics (and science).

Prior Research

The activity of reading graphs has traditionally been framed as "interpretation" which is taken to be the "action by which a student makes sense or gains meaning from a graph (or portion of a graph)" (Leinhardt et al., 1990, p. 8). These authors further suggest that depending on the referent of a graph, interpretation locates meaning in some naturalistic situation or in the same mathematical space. The movement required by an interpretation into a different modality of representation has been termed a "translation" (Janvier, 1987; Kaput, 1987). While from the phenomenological and semiotic perspectives taken here all reading involves interpretation and translation, we are most interested in translations that involve the relationships of graphs and situations, and, necessitated by the activities we observed, those between graphs and other graphs. It is important to keep in mind that the situations to which the graphs refer¾ for example, the density dependence of birth rate¾ are themselves embedded in other situations, in this case the lifeworld of an ecologist or a psychological laboratory. In a study of mathematical practices in a grade 8 science class we showed that this second kind of situation codetermines the level to which a graph can be considered as "contextual" (Roth, 1996).

In previous research, much attention was focused on situations involving moving objects and therefore graphs in which position (distance), velocity (speed), and acceleration are plotted versus time (Berg & Philips, 1994; Clement, 1989; Mokros & Tinker, 1987; Nemirovsky, Tierney, & Wright, 1997; Roth, 1993; Roth, Tobin, & Shaw, 1997). Other tasks which have figured much less prominently include heat and temperature (e.g., Linn, Layman, & Nachmias, 1987), interaction of biological and physical variables (Preece & Janvier, 1992; Roth, 1996), and interaction of biological variables (Bowen, Roth, & McGinn, 1999; Roth & Bowen, 1999a). In the present study, we focus on graphs that are used in the domain of ecology and therefore relate to situations (although some scientists related these graphs also to other graphs), and display relationships and interactions including biological and physical variables.

Relative to translations between graphs and situations, a review of the literature (Leinhardt et al., 1990) identified two major difficulties students experience: slope/height confusions and iconic interpretation. The first type of difficulty arises when students are asked about the relative speed of two objects in the context of a graph that displays distance-time information. Rather than identifying the relative speed (from the slopes of the curves), most students compare the relative heights of the two curves (often lines) presented. Iconic interpretations include all those errors where students inappropriately relate a topological feature in the situation (e.g., a curve on a race

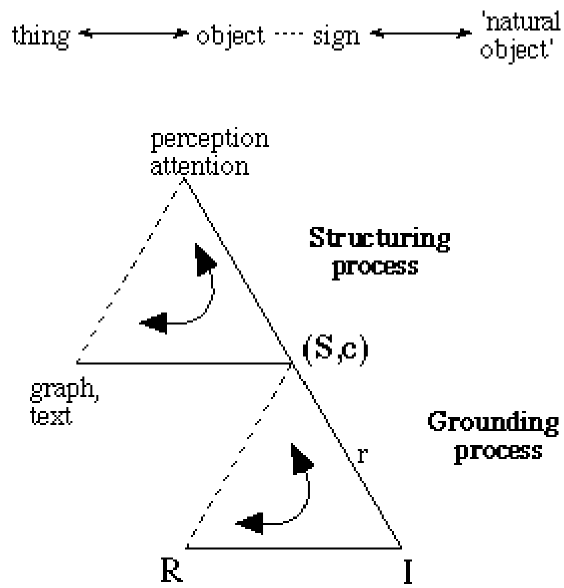

Figure 1. Two stage, semiotic process of graph reading. On the upper left hand side, the perceptual analysis S’ carves possible sign objects I’ from the text R’. The output of this process then become the signs S which point to, or are grounded in, a possible world R as described by the interpretant I (lower right hand side). The interpretants I are constrained by a sign’s context c and conventional constraints r on sign use.

track) and a similar topological feature in the corresponding representation (e.g., a bend in a line graph). In the context of the theoretical framework underlying the present study, both types of difficulties are to be expected and were in fact observed even among scientists when they deal with graphs that, though from their own domain, were (relatively) unfamiliar.

As a result of previous studies, we suggested that (a) Cartesian graphs establish virtual spaces and traces (Roth, Bowen, & McGinn, 1999) and (b) that to interpret graphs means to build rich situational descriptions from reductionist and transformed mathematical representations (Roth, 1996). Thus, there are two fundamental difficulties presented to the interpreting individual. First, because graphs, as other sign forms, have arbitrary but conventional relations to the things they represent their content cannot be elaborated without knowing these conventions. (In this article, we remain consistent with the historical development of semiotics and use the word “sign,” although in the mathematics education literature “symbol” has also been used to denote the same objects [e.g., Kaput, 1987].) Second, because graphs contain little circumstantial information (Preece & Janvier, 1992), building a description of a situation from which the graph might have come is a task inherently underdetermined by what information is immediately available in the graph. As a result of our research among scientists, we developed a semiotic model of reading graphs (Roth, 1998).

Semiotic Model

Our research is based on the semiotic premise that graphs and their written or spoken subtexts are multimodal texts (Bertin, 1983; Eco, 1984; Lemke, 1998; Saint-Martin, 1990). Each text is constituted by matrix of sign elements; both individual sign elements and entire texts are subject to the semiotic efforts of those who read them (Eco, 1976; Ricœur, 1991). Our model of reading graphs contains two component activities of each obtaining a dialectic nature: structuring and grounding (Figure 1). In structuring, the reader carves the graph and subtext to construct relevant signs; here, readers use known graphs and texts to constrain and elaborate perceptual and attentional processes. In grounding, readers link these (resultant) signs to material objects or other signs; here, familiar objects and sign systems readers constrain and elaborate the sign itself. Reading itself, and therefore the two component processes, is situated in the sense that it is normally part and contextualized by other activities in which a person engages. We begin our discussion of the model with the grounding process because it is that part normally considered in the literature on graphing.

Grounding

Our model of grounding is based on the relationship between a sign S, referent R, and interpretant I. A sign, is one (material) entity that refers to and stands for another. A referent is an entity or state in the world to which the sign refers to and stands for. An interpretant is another sign that stands for or elaborates the relation of sign to referent. (According to the Handbook of Semiotics [Nöth, 1990], there are many models of sign use. Our version makes use of a triadic relationship that was developed independently in structurally equivalent form by Frege [1892], Richards and Ogden [1923], and Peirce [1931].) For example, in the population

In the derivation of a logistic model, we assume that, as N increased, birth rates decline linearly and death rates increase linearly. Now, let's assume that the birth rate follows a quadratic function (e.g., $b = b_0 + (k_b)N - (k_c)N^2$), such that the birth and death rates look like the figure. Such a function is biologically realistic if, for example, individuals have trouble finding mates when they are at very low density. Discuss the implication of the birth and death rates in the figure, as regards conservation of such a species. Focus on the birth and death rates at the two intersection points of the lines, and on what happens to population sizes in the zones of population size below, between, and above the intersection points.

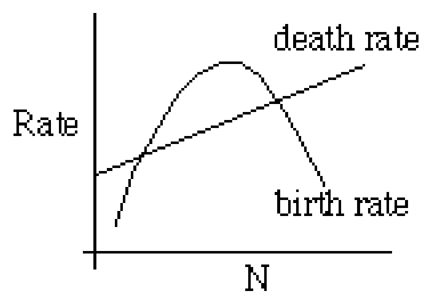

Figure 2. One of the tasks presented to the scientists. It was one of the problems given to students in a second-year university introductory ecology course.

graph (Figure 2) which presents birth and death rates versus population density N, a reader may perceive as relevant one intersection of the two lines and constitute it as sign S_1. This sign S_1 will be read in the context, c, of other signs. Here, she may constitute c in terms of the signs “birth rate” (S_2) and “death rate” (S_3). As a consequence, the reader may then produce the interpretant I_1, “birth rate equals death rate”; as structurally equivalent interpretant $I_{1'}$ she might produce “$b[N_0] = d[N_0]$” or “$b[N_0] - d[N_0] = 0$” after having identified N_0 as that value of the abscissa where the curves intersect. Thus, “the relation of a ‘sign’ to an ‘object’ is such that another relation, that between ‘interpretant’ and ‘sign,’ can be grafted onto the first” (Ricœur, 1991, p. 123). The process, however, does not have to stop here, but the interpretant I_1 can itself become a sign which the reader further elaborates by the interpretant “the population density does not change” (I_2) which also replaces or translates the original sign. This new interpretant may be associated with some state in the world where the population density does not change, for example, the wolf population in Glacier National Park. Here, signs are elaborated by one or a series of interpretants and set into a relation with the sought-for referent.

Our previous research shows that the process of grounding signs in the world (relating S to R) is inherently dialectic (Roth & Bowen, 1999a). That is, sign-referent relations are the results of two mutually constraining moments: a synthetic construction of possible referents from the signs and an analytic reduction of familiar situations into signs. For example, “birth rate decreases with increasing population density so we should see less children being born when there are more people” constitutes a movement from sign to referent (S—>R) whereas the statement “In India, birth rate remains high despite increasing population density so the birth rate curve should stay up” constitutes a movement from referent to sign (R—>S). In the dialectic production of these types of statements in the course of a reading, individuals reify both sign and referent, and thereby disclose meaning and unfold understanding.

These interpretants, to be consistent with cultural norms, necessitate that each sign is used according to conventions, and that other conventions, r, such as “right on abscissa” and “up on ordinate” signify an “increase” in frequency or “higher” values. This is not always the case and represents a source of potential difficulties for a learner. For example, “national debt” may be shown to increase by means of a curve with a positive slope. Yet in spreadsheets (accounting, banking), debt is associated with a minus sign so that “increasing debt” should be plotted, to be consistent with other mathematical conventions, as a curve with negative slope. Finally, and consistent with a radical constructivist approach, there are no ideal referents but always and already formulations of referents in some sign form, frequently vernacular (Derrida, 1988; Eco, 1984). The process of grounding can therefore be understood as a process in which the reader of a graph establishes a relation (not equation!) for the quadruplet {R, S, c, r} of the type $R = f_r(S,c)$ (e.g., Barwise, 1988). By producing interpretants, readers may generate a new sign that replaces the original sign such that it allows them to more easily solve the relation.

Structuring

The task of the graph reader therefore is to identify relevant signs and construct interpretants that ultimately describe some state in the world. This state can be a situation or another sign (Becker & Varelas, 1993). When individuals are familiar with the signs they are reading (e.g., the morning newspaper, a journal article from one's domain), the signs become transparent and reading leaps beyond the sign to the signs signified. The mathematics educator does not have to interpret or analyze the sign "graph" but, upon seeing the word, instantly "knows" what its possible referents are. Without difficulties, she can more or less identify instances which are (not) appropriately designated by the sign "graph." However, when signs or sign complexes are less familiar, reading requires the engagement in an activity where the structure of the text and text-internal relations have to be disclosed in the process of reading. For example, artistic images, research reports from another domain, or musical scores are sign complexes that do not allow most people access to their respective referent domains. Often, whether or not some aspect of a graph should be taken as a sign or simply as an incidental feature is unclear; readers are then involved in a structuring activity where the very nature of a graph's signifying aspects is unclear (Roth, 1998). That is, it is not immediately evident whether readers of Figure 2 should construct as a relevant sign the one or more of: relative height of the two graphs, slopes or relative slopes, intercepts, intersections, maxima, and so on. Our transcripts also show that not all of these features are salient to the readers all of (or at any) time. This process of structuring a graph and its caption is represented on the top part of Figure 1. Here, the outcome of structural analysis of the graph/text produce interpretants (Ricœur, 1991). These interpretants, specific features of the graph, then become the signs for the second process which relate signs to things in the world (bottom right). To reiterate, our data show that when graphs are unfamiliar, even our scientists do not know a priori how to structure a graph so that it can become a sign that points to something beyond itself.

Reading is made difficult by the fact that the structuring processes themselves are a function of familiar signs and prior understanding, for it is well-known that perception is theory-laden (Feyerabend, 1975) and that the physiological processes regulating attention and perceptions are functions of prior experience and understanding (Jarvilehto, 1998). The following analogy may be helpful for understanding the two processes of structuring and grounding. Thus, graph reading is an activity not unlike that in which Sherlock Holmes engages or the protagonist of Eco's (1984b) *The Name of the Rose*. Sherlock does not know a priori whether some cigarette butt refers him to the criminal or not; initially, he may not even notice a cigarette butt such that is does not exist in his lifeworld. Thus he engages in structural analysis to separate signs (figure) from ground; he then relates the ensemble of signs to events in the world. (Here, signs and events are entities of the same logical type.) In the course of a novel, our protagonist may have to restructure his perception of a situation such that something that he first constructed as a sign does not fit in his unfolding understanding of the events that led to the cigarette butt being where it is. He may have to construct new signs (structuring), or relate already constructed signs to new contexts c (e.g., brand of the cigarette), understand signs from within different sets of conventions (within a transvestite culture, there are cigarette butts with lip stick that were originated by a male), or construct additional interpretants (e.g., the butt as a sign originating from an unrelated event).

Research Design

Participants and Data

The present study is part of a larger research agenda designed to understand graphing from middle school to professional practice (e.g., Roth & Bowen, 1999c). As part of this research agenda, we are particularly interested in understanding the competent reading of graphs and the kinds of experiences it requires. Over a two year period, we therefore included information from the following sources.

Sixteen practicing scientists who had obtained an M.Sc. or Ph.D. and who had a minimum of 5 years of experience in independent research were asked to interpret the same set of three graphs. In addition, they read one or more graphs which they provided from their own publications. The sessions lasted between 1 and 2 hours. To better understand changes in the understanding of graphs, we conducted a 2-year ethnography in an ecological field camp. We accompanied (and sometimes videotaped) the field work of our primary informant, a herpetologist, and conducted interviews with the field researchers. These data were enhanced by 10-hours of videotaped interviews on the main campus during the winter¾her non-data collection season. The interview questions related to the transformation of data and production of her own graphs, and the reading of additional graphs related to her domain but with which she was not very familiar.

To better understand the reading practices related to familiar graphs, a water technician (Karen) was videotaped on four occasions reading pen chart-recorded graphs from her work (a) in real time at the work site (twice) and (b) during a public exhibition of her work as part of an open house organized by an environmental activist group. Additional information about the context of Karen’s work was gathered as part of an ethnography covering this activist group. This included accompanying Karen on trips through the watershed, building rock structures in the creek that oxygenate the water (“riffles”), and inspection trips of habitats in and along the creek.

Tasks

For the interpretation tasks, we chose graphs as they appeared in the introductory ecology course. Our textbook analysis showed that these graphs are fairly typical: With minor variations, there were 120 graphs of the same three types in a popular 800-page introductory textbook (Ricklefs, 1990). One of these graphs was a population graph (Figure 2) in which birth and death rates are displayed as a function of population density. Figure 2 constitutes a graphical model which, in the field of ecology, came into use in the 1950s and 60s (Kingsland, 1995). There were 60 such graphical models in the introductory textbook. The two other graphs displayed (a) three elevation and climate-dependent distributions of plants that are distinguished by their different photosynthetic mechanisms and (b) three isographs portraying different scenarios of the interaction of two resources on a third variable (plant growth).

We interviewed the professionals either in their own offices or in the office of the lead author. Although we presented the tasks as ones which were part of our interest in people’s differing interpretations of graphs, our participants often treated the tasks as puzzles to which someone (we or some indefinite author of the graphs) knew the answer. The graphs brought to the interviews by the scientists usually displayed either data of correlational nature or those which were collected from experimental designs. There was a shift in the orientation toward the task when scientists talked about their own work to provide us with a reading of a graph. At this point, they entered the familiar worlds of their everyday work and, for some, the objects they had studied for several decades. We actually drew our data on the water technician Karen from a context which is part of her work. (All proper names and details of lifeworld situation have been changed sufficiently to protect the identity of the participants.)

As a water technician, Karen produces graphs with a pen chart recorder that monitors the water levels of a creek running through the farm where she is employed. Three years of these graphs also contained information that she had added by hand¾ the amount of rainfall per day throughout the year. The current year's graph was directly pulled from the pen chart recorder so that her reading was produced concurrently with its generation. Karen frequently reads and explains these graphs to visitors of the farm or to visitors of open houses organized by an environmental activist group that also works closely with Karen and her employer. We recorded Karen in four such situations which are therefore circumstances in which her interpretation is provided in "authentic" settings as part of her everyday activity of using the graphs.

Data Processing and Interpretation

All videotapes were transcribed in an ongoing manner so that the text was available in written form during our analysis. We conducted analyses both individually and collectively. Our analyses, grounded in semiotics and hermeneutic phenomenology, are based on the assumption that reasoning is observable in the form of socially-structured and embodied activity (Garfinkel, 1991). In our analyses, videotapes, transcripts, and artifacts produced by the observed individuals are natural protocols of their efforts in making sense of, and imposing structure on, their activities. (Of course, part of this protocol relates to activities that bear little relation to our participants' daily work whereas reading their own graphs is similar to other situations in which they explain their work to interested lay people.) These protocols constituted our texts that we structured and elaborated in our analyses.

We independently read all transcripts and viewed all videotapes before meeting for collaborative analysis. We conducted extensive collaborative analyses of lectures and interviews following the precepts of Interaction Analysis (Jordan & Henderson, 1995). Videotape replay was stopped whenever one of us thought a significant event had occurred. This person then stated an assertion before the event was reviewed as often as necessary for a full exploration by both researchers. We subsequently reviewed other episodes to check the degree to which they confirmed or disconfirmed the assertion. On the basis of these checks, we reformulated initial assertions until they were representative of the data. We then discussed personal constructions, subjected them to critique and analysis, and tested them against the entire data set to evaluate fit and plausibility. Our model therefore emerged as a grounded theory from many such iterations.

Because we are trained as natural scientists (graduate majors and/or minor in: physics, physical chemistry, statistics, applied mathematics [Roth]; ecology and sociology [Bowen]), there exists a possibility that we misread the data because of our status as "natives." To deal with this possibility, we followed the advice of hermeneutic phenomenology to produce alternate readings of the data and thereby engage in a dialectic of understanding and explanation (Ricœur, 1991). The ecologist read the transcripts in the light of his own graph-related learning process during his undergraduate and graduate training; the other engaged in hermeneutic and mathematical structural analyses.

Professionals Read Graphs

When we began our research on scientists' interpretations of graphs we were mainly interested in collecting several interpretations which we could use as a kind of standard in our work with middle school to university students. However, we noted very quickly that (a) the interpretations varied considerably across scientific disciplines (ecologists versus physicists), sub-disciplines (theoretical versus field ecology), and special interests (conservation versus harvesting); (b) there existed a broad range of competencies including a substantial number of scientists who struggled with and even abandoned the task; and (c) the nature of the task seemed to change when scientists interpreted graphs from their own work.

To better understand how professionals read familiar and unfamiliar graphs, we produced two detailed case studies because it is through the particulars of unfolding graph interpretations that we come to better understand the conditions of expertise (e.g., Moschkovich, 1996, 1998; Schoenfeld, Smith, & Arcavi, 1994). These case studies show how graph reading unfolds in real time, and therefore have all the "mumbles, stumbles, malapropisms, metaphors, tics, seizures, psychotic symptoms, egregious stupidity, strokes of genius" (Rorty, 1989, p. 14) characteristic of cognition in real time.

We present our materials in three sections. In the first section, we use a case study to illustrate problems in the reading of one scientist, Ted. (Six of the 16 scientists struggled even more than Ted in situations where they were not familiar with graphs and which came from contexts other than their own work.) In the second, we use the case study of the water technician Karen to exemplify transparent reading and the kind of knowledge and understanding that accompanied competent graph reading; this reading is typical for situations where scientists read graphs that they generated and used as part of their everyday work. In the third section, we contextualize differences when scientists interpreted our graphs and those from their own work.

Reading an Unfamiliar Graph

Though some graphs may represent standard fare in the undergraduate training of a scientist he may not be familiar with it or the phenomenon that it is said to be about. The activity of reading graphs is therefore different from those that characterize everyday work and life. In this situation, the task of interpretation is clearly twofold: people have to reconstruct the internal dynamics of the multimodal text, and have to restore to the text the ability to project itself outside in the representation of a world that we can inhabit. This double task is represented in the two sides of our model (Figure 1). To better understand this task, we present considerable sections from the reading of the population graph (Figure 2) by one scientist, Ted.

Ted had studied physics (Ph.D.) and, for the past 15 years, has been an active and internationally-renowned scholar in a government research unit. Ted regularly publishes research articles (2-4 per year) that include Cartesian graphs. In the course of his reading, Ted marked up the population graph and added his rendering of what the population dynamic would be over time given different population sizes as the starting point (Figure 3). Compared to the field ecologists in our data base, his interpretants included more references to mathematical processes and descriptions and fewer references to actual animal or plant populations. In this way, his reading was more like that of the theoretical ecologists and the other physicist in our data base. He frequently drew on resources from everyday life to elaborate interpretants.

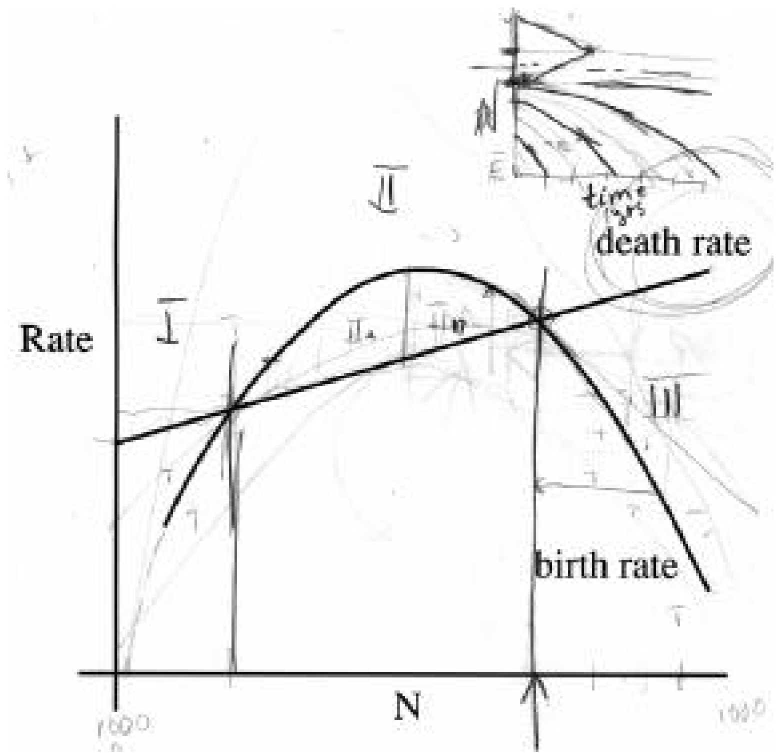

Figure 3. Graph in problem set marked in the course of his reading by one scientist. Notably, he produced his own graph representing the development of population size over time in the different regions he had identified.

<u>"They are obviously plotting some population. . ."</u>

As his interpretation unfolds, we can see Ted engage in the processes outlined in our model. Furthermore, when encountering difficulties, he enacts readings that are analogous to difficulties identified among high school students. Unlike most of his peers, Ted does not begin with the caption, but engages in a reading of the graphical representation and the textual information within it before reading the caption. (Roman numerals are used to number utterances.)

> [i] N would be the number like the individual in a population, rate would be a number differentiated by time, so this would be a measure of change. [ii] They are obviously plotting some population where we take, find the number of individuals and we see that as the number of individuals goes up the rate, the death rate increases. . . [iii] There are more individuals dying per unit time as the number goes up and birth rate increases as the. . . [iv] It probably should go to zero if there are no, well it should go to 2 probably; if there are no parents, there will be no births. [v] The birth rate increases to a maximum, at some optimum number and then the birth rate falls of as the number of individuals increases, [vi] probably because of limits in the environment or competition or disease or overcrowding or social problems within that population. [vii] Now, the logistic model is a mathematical formulation and it's very common I know in the ecological literature and this particular curve POINTS[birth rate], logistic curve, has been shown, it has been shown or has gained a lot of notoriety because it demonstrates chaotic behavior in non-linear. . .

In this opening reading, and without reference to the caption, Ted immediately elaborates the signs "N" and "rate" with the interpretants "number of individuals in a population" and "number differentiated by time," respectively [i]. Both interpretants are not immediately available from the task, but arise from common conventions (r) with which more theoretically-oriented scientists are familiar. Ted then enacts a literal reading (i.e., describes graph as graph but in words), and produces an interpretant which translates the trajectory of lines with respect to the coordinate grid into a verbal representation [ii], which he then further elaborates in [iii] with a second interpretant. All of a sudden, Ted engages in a reverse movement where he takes everyday understandings of reproduction (no organisms no births, it takes two parents to procreate) to project where the birth rate curve should intersect with the abscissa [iv]. In this, we observe a movement from the referent R to the sign S. Ted continues to provide interpretants that describe the shape of the birth rate curve [v], and then again uses familiar understanding of ecology (R) that legitimize the dropping of the birth rate (S) with increasing N [vi]. Here, the situation descriptions "limits in the environment," "competition," "over crowding," and "social problem" are not available in the graph or in the caption. Yet here, Ted's understanding of ecology is consistent with his interpretant of the birth rate curve for larger N. Therefore, the ecological understanding (and prior experiences) and his reading of the graph reify each other (double arrow in the bottom right triangle). Ted then turns to the text and reads the term "logistic model," leading him to relate it to mathematical formulations and chaotic behavior in non-linear systems [vii].

<u>"At intersection points you are at equilibrium. . ."</u>

Up to this point, Ted engaged both in internal process of structuring the graph by producing interpretants (e.g., birth rate drops) which then became signs that project beyond themselves to a possible world. We also observe the reverse movement where descriptions of the world as it is taken to be (e.g., competition, overcrowding) are used to reify and legitimize the shape of the birth rate curve.

> [viii] At intersection points, presumably, you are at equilibrium, the number of individuals in the population, they are being replaced as quickly as they’re being removed and the same will hold at the other equilibrium point. [ix] In between, the population is increasing in size and if you’re concerned with harvesting this population for some reasons, hunting or trapping or fishing, you would want to make sure that you’re keeping your population numbers in this range. [x] The most increase you have is here POINTS[b - d = max] because you calculate the population, say the P is equal to the initial P plus birth rate plus death rate.

Ted individuates the two intersection points [viii]. This in itself is significant in the light of the fact that in a different graph, he (as other scientists) does not highlight the existing intersections. Thus, the intersections become salient (or are made salient by the caption) not in themselves, but in the light of other signs that the reading individuated (which therefore constitute the context c of the intersection sign), “birth rate” and “death rate.” Furthermore, for the intersection to be significant, the graph relies on the reader’s adherence to the conventions (r) regulating the use of the two signs “birth rate” and “death rate” as opposite trends that tend to increase and decrease an existing population. This is even more evident when we compare this graph with its copy where the two labels are replaced by “birth rate of deer” and “birth rate of moose.” In this case, the implications of the intersections are of a very different nature. Thus, in one of the other tasks Ted, as his peers, does not make salient the intersections of the distributions of different plant types.

Ted’s reading of the intersections as points of equilibrium therefore presuppose complex interactions between extant (socially-shared) understandings of how the world operates and the graph at hand. His interpretant “equilibrium” for the sign constituted by the intersecting curves is legitimized by a further interpretant of “birth rate equals death rate” in the form of “they are being replaced as quickly as they are being removed” [viii]. Ted’s next statement [ix] shows again a complex interaction between knowing the world and the topology of the signifying graph. Here, in the context of the conventions regulating the use of “birth rate” and “death rate,” the relation between the two curves suggest an increase in number of individuals in the referent population. Conversely, “hunting,” “trapping,” “fishing,” and “harvesting” are from the domain of the experienced world. His ecological common sense suggests that to engage in these activities, sufficient population sizes are required unless the species is to become extinct. According to Bourdieu (1980), common sense (“sens commun”) itself is the result of interactions between our intuitive and tacit understanding of how the world operates (“sens pratique”) and how we represent the world in everyday language (“sens objectivé”). Ted uses such a common understanding to suggest that activities leading to the density-independent reduction of population size should be encouraged when the population tends to increase its size. This interpretation is only partially correct, however, as it does not include the region for population sizes above the second intersection. Ted develops this non-standard interpretation even further in subsequent statements.

The last interpretant in this section relates to the maximum difference between birth and death rate curves [x]. As he points to that part of the graph, he elaborates it in terms of the maximum increase in the population size and provides an algebra-based rational. This, however, is a non-standard interpretation for the maximum increase in individuals occurs when the function f = (b[N] - d[N])·N is maximized rather than when (b[N] - d[N]) is maximized. Here, Ted does not attend to or observe the convention of interpreting birth rate and death rate as the increase and decrease in the number of individuals relative to the population size. This non-standard interpretation is not unlike the slope/height difficulty observed among students reading a distance-time graph, for in both cases the interpretant is based on some maximum in the graph currently available rather than in a transformed graph. In both instances, situated reasoning (inappropriately) associates the utterance of "most" (or faster) with the differences visibly at hand rather than with the maximum differences in the associated f[N] or velocity-time graphs. The fact that velocity can be read from a position-time graph is due to the fact that, given the definition of velocity as $v(t) = \frac{dx(t)}{dt}$ happens to coincide with the derivative x' of the x(t) function. The conceptual definition of a quantity coincides with the derivative of another quantity plotted in a specific way. Representations of x as a function of any other variable would not support similar inferences about the velocity.

"So you have a family of curves. . ."

In the subsequent episode, Ted continues by shifting his attention to the relationship between birth and death rates to the right of the second intersection. In the process, he begins drawing his own graph in which the population size is represented as a function of time (top right in Figure 3). In this episode, the drawing progresses to the point where a family of downward sloped curves intersect at various points with the abscissa.

> [xi] If we're working out in this range, the birth rate is much lower than the death rate, so the number in the population versus time which would be the other curve that's behind this one will show a declining population. [xii] I believe that if we looked at the population versus time, in year say DRAWS[graph], if we were in, I call this Domain I, Domain II, Domain III, we are in Domain III LABELS[sections] because the initial population sitting up here is dying at a rate greater than it's been born, then over time you would have a decline in the population. [xiii] It wouldn't be strictly linear because this is a power curve and this is linear. So you would have a convexity or a concavity to it, I'm not sure. I think it will be like this, down, because if we took a tangent at this point, this would be a linear response and as the numbers get higher, you're actually getting. So, you have a family of curves that all do this DRAWS[curves].

Here, Ted produces the interpretant "declining population" for the sign constituted by a birth rate curve lying below the death rate curve [xi]. At this point, it is not clear how far the population may decline. However, after sectioning the graph into three domains (Figure 3, [xii]), Ted draws a family of curves representing the development of populations over time giving an initial size from Domain III (right of the second intersection). The curves in this family intersect with the abscissa and therefore represent crashing populations. This interpretant would have been standard if birth and death rates were not functions of population density (or size) but constant. This non-standard interpretation was provided by 37% of the scientists and by 78% of all non-scientists.

<u>“. . . that’s when you get in this chaotic thing. . .”</u>

As Ted elaborated on his earlier interpretants, there arose the opportunity to individuate this problem.

> [xiv] If you were at these points here, birth and death rates, you would have a family of, we have, this in here, I call this, I should call this N, whatever this value is. [xv] You have a straight line across here, versus time, and this value which is a little higher, you’d also have a straight line versus time, it’s an equilibrium. [xvi] And if you get an N that’s higher than that, then it would appear that that is going to drop, to drop off. [xvii] But then you, that’s when you get in this chaotic thing, it’d fall back down in this zone and it would be an equilibrium again I guess.

Here, as he returns to include Domain II in his own graph, he draws lines horizontal to the abscissa each representing one of the two equilibria [xiv]. However, conventional wisdom in mathematical analysis holds that these lines cannot intersect with the family of curves so that he draws them directly above. When he then draws a new curve beginning with an N above the equilibrium [xvi], his line stops at the upper horizontal line. Apparently without conflict, he continues in his interpretation. This conflict might have been expected given that he earlier had drawn a family of curves starting for N’s above the equilibrium point but which terminated at the abscissa. The final interpretant [xvii] is somewhat cryptic at this point, but becomes clearer below when he explains that from his understanding of natural populations, he expects oscillations. We also find an allusion to the “chaotic thing” which he associated earlier with the behavior of non-linear systems and the “logistic curve”; the latter was also an interpretant of the quadratic birth rate function [vii].

<u>“. . . and talk about slopes of convergence”</u>

In the next episode, Ted then develops the “chaotic thing.” As his subsequent elaborations [xx] indicate, he is searching for something in the original graph [xix, xxi] that would lead to an oscillation, an instability, of his own curve around the equilibrium.

> [xviii] I never worked with a logistic curve. . . But if you were up, depending whether you’re inside that zone or outside that zone and whether you’re on this part of, and we can call that Zone IIa and IIb LABELS[zones], because you’re diverging from the death rate and here you’re converging toward it, you probably, you could probably divide this one into an upper and lower portion and talk about slopes of convergence toward the equilibrium within that. [xix] I should be able to come up with some way that would go unstable but I don’t see it right now. [xx] By unstable I would mean something that would, an oscillation that would put you sometimes above this curve and sometimes below it so that you didn’t converge to this equilibrium. [xxi] That would be to me an unstable population and I don’t see how these two curves, I’m not seeing it right now anyway.

Despite engaging in a focused perceptual and structural analysis of the original graph, Ted does not find a feature which, qua sign, would lead to an interpretant consistent with an oscillating (unstable) population. Here, we observe a process where the unsatisfied expectations of a scientist drive him to question his original reading. That is, from the expectations of population oscillations based in his understanding of the world, Ted not only returns to the level of the signs, but to the structural analysis of the graphical text which no longer stands in a mutually constitutive relationship with the phenomenon. The activity is not unlike that of a person looking for the first time at what appears to be a random collection of black spots on white ground and finding out only after some time that one can perceive a Dalmatian or a cow (see Churchland, 1995 for a discussion of the neuroscientific explanation of such structuring). At this point, the graph is therefore a highly ambiguous text requiring an analysis not unlike those of esthetic texts (e.g., paintings, music). In fact, his comment “I don’t see it” lies on the reverse movement from text to structural analysis (see arrows in Figure 1) questioning the process of perception itself which lies at the origin of the graphical text as a structured object.

His analytic process turns up differences in the relative slopes of the two curves in the middle zone which he sectioned off into two subzones. Thus, the relative slopes heretofore absent in his analysis become part of the resources brought to his interpretation. We can say that his ontology, the ensemble of the signs in the task world, has changed. Without elaborating the point, the interpretant [xxviii] raises the possibility that the instability comes from the different “slopes of convergence,” the fact that the two lines converge and diverge in the two subsections, respectively. Of course, this approach is not unlike that described as iconic difficulties students face. Convergent and divergent lines in one graph are treated as isomorphs of the expected convergence to and divergence from the equilibrium so that it could not be reached. This becomes evident in the final episode discussed here.

<u>“. . . that’s what I mean by looking for some kind of instability.”</u>

Ted seeks a feature in the graph such that the curves in his own graphs never reach the lines representing the equilibrium state, but oscillate about them [xxii].

> [xxii] If you use it in a management sense you’re aiming for a particular population estimate and what you find, certainly what is always there, all of the case examples of population management by man occurs that there’s always overshoot, undershoot and feedback loops that prevent that population from ever reaching these theoretical equilibrium points. [xxiii] And that’s what I mean by looking for some kind of instability in this relationship that would. . . and it could be that it has to do with the. . . [xxiv] If you change these coefficients k_b and k_c you change the shape of this curve. . . death rate is linear, and if it were not, it may be, it’s also, has a logistic model shaped to it, maybe consider this.

Although he did not pursue the implications of the convergent and divergent slopes, Ted moves on to individuate other aspects of the graph which heretofore were in the background, or did not exist at all from his perspective. His ecological common sense provides a strong suggestion that the representation (sign) should oscillate. As this is not the case, Ted returns to the issue and explores ways of perceiving the graph. As part of the process of perceptual restructuring, he turns up new potential signs [xxiv]. In the light of an earlier interpretant, that the logistic model “has gained a lot of notoriety because it demonstrates chaotic behavior in non linear systems” may introduce the instability which Ted’s interpretation attributed to natural systems. He ultimately abandons his efforts without further elaborating on the issues raised so far.

In summary, the episodes provide glimpses of a scientist attempting to read the graph with the purpose to leap beyond its material basis and to relate it to the world he knows. The graph is unrelated to what he does in his everyday life: the activity is more akin to completing unfamiliar tasks "indoors" (Lave, 1988; Roth & Bowen, 1999c) rather than constituting "cognition in the wild" (Hutchins, 1995). This reading is not effortless and in some instances is non-standard because (a) standard conventions were not observed, (b) structural analysis produced individual and ensembles of signs which did not or were not used to constrain their respective interpretants, and (c) visual features of the graph were used to make inappropriate structural arguments. We also saw an interaction between structural analysis and understanding of how the world works, and dialectic relations between sign and referent dimensions in each of the two processes at work. The video in our data base show that (a) 6 of 16 experienced scientists experienced even more trouble than Ted with this graph and (b) much more than do non-scientists, scientists generally connected their reading of this graph to their understanding of the world or mathematics. The difficulties faced by scientists with these provided graphs starkly contrast with their reading and use of graphs from their own work.

Reading Transparently

Although most of our videotapes show scientists reading their own graphs, the domain-specific discourse is so highly technical that they would require many explanations about the biology, biochemistry, physics and so on of the phenomena and theories at work. As our model predicts, a lack of this knowledge will make it difficult to impossible for the reader of this paper to understand the graphs themselves. We therefore selected the case study from a domain that does not presuppose a scientific training: A water technician (Karen) reads graphs (Figure 4) produced by a pen chart recorder monitoring the water level of a creek flowing through a forest-rimmed valley, spotted with farms. Karen works for Ocean Side Farm whose owner, as Karen, participates in various environmental action groups interested in securing the health of the local watershed. Karen produced the graph readings as one part of her work which included public relations for the activities of the environmental action group and the farm, who are both engaged in contributing to the ecological health of the watershed. We return to the significance of this in next section.

The graph readings presented here typify those we recorded when scientists read graphs from their own work. In all instances, the graphs were transparent to the act of reading and the interpretants seemed to leap beyond the material basis of the sign to the things they described. Furthermore, these individuals could produce a sign standing for an actual or hypothetical state in their familiar worlds.

"In the summer, we're down here. . ."

In the first excerpt, Karen and a group of interested environmentalists and teachers stand next to a small water monitoring station. From the pen chart recorder that is visible inside, Karen has pulled about one meter of chart with a graph representing the water level of the creek over the past few days. While pointing (Arabic numerals index the places she points to in Figures 4.a) to different places on the chart, Karen reads the chart including a part that she had not seen before. The excerpt begins after she has provided an indication of the scale and explained that to obtain the water flow volume, the chart has to be read in terms of a calibration curve which is not ready to hand. (Roman numerals are used to number utterances, Arabic numerals to cross-reference the places on the graph that she pointed to.)

> [i] In the summer, we’re down here [1]. [ii] We’re about, the first square [2] doesn’t mean anything, that’s just a bit of lee-way, we’re at about two squares up. We have about 12 liters per second. [iii] In a really, really incredible rain event, we get right to the top here [3] and it’s 5,000 liters per second. [iv] So, we have a creek that really increases its volume, as the smaller creeks really fluctuate. [v] And then we have all of our storm drains in the watershed funnel into the system. [vi] So, on top of the rain fall event, from the areas along side the creek and in the creek itself, we get all this water augmenting this flow. [vii] So, this peak [4], which maybe hypothetically, 20 years ago would have come up to 3000 liters per second [5]. [viii] Now after a rain fall event, it’s coming up to 5000 liters per second because we’re getting the run-off from all the pavements, rooftops, roadways, and we’re getting all the contaminants with that as well. [ix] If that low [6] in the summer gets below that first square [7], we dry up, farming cannot occur and a lot of farmers depend on the creek.

This episode begins with Karen projecting a state in the world, “water level in the summer,” onto the chart [i]. That is here, beginning with her understanding of the world, and from past experience with the chart, she enacts a reverse movement from the referent to the sign (graph).

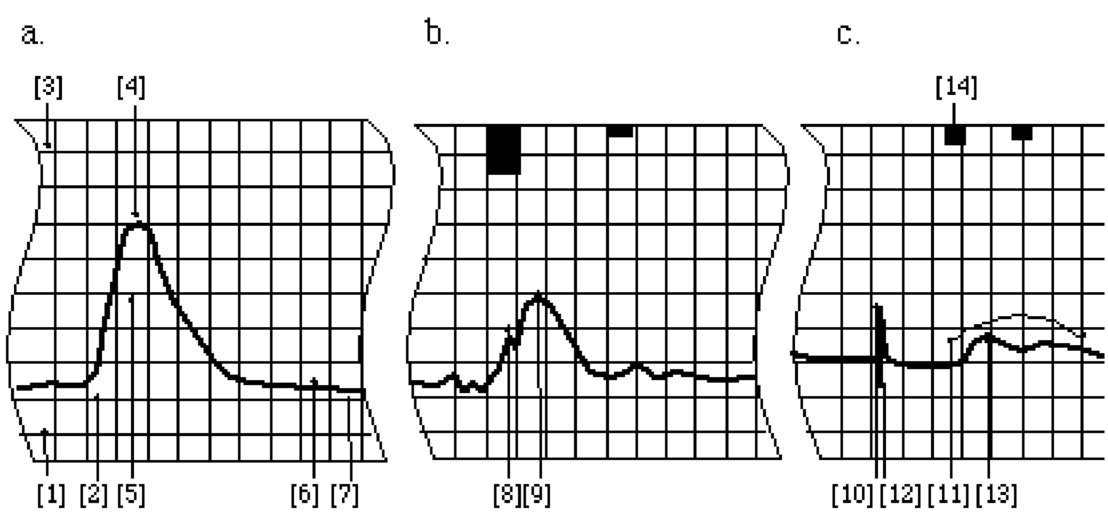

Figure 4. Three sections from four years of pen chart records. a. Section pulled out of the recording device and read on-line. b., c. Two sections read to visitors of an open house related to the ecology of the local water shed. Here, the amount of rainfall as recorded in a separate device was plotted on the upper part of the chart. Numbers refer to places pointed to during the reading.

She then elaborates the previous statements by translating “down here” into a measure of flow (volume/time) [ii]. In the next utterance, we observe again a movement from the referent domain, the experienceable “incredible rain event,” to the sign domain and into another sign domain: by indicating where she would expect the curve to come lie on the paper (S_1) and the amount of flow in the creek thereby indicated (S_2). In the next three utterances, Karen provides situation descriptions from the referent domain [iv-vi]. She talks about the creek and its smaller, fluctuating tributaries, storm drains that funnel into the watershed, and the water from the areas alongside the creek all of which increase the water flow to the earlier described levels. That is, she does not merely read or project a graph, but has an intimate knowledge of the situation and the conditions that bring the water to the high levels of 5,000 liters/second. Not only that but Karen is also familiar with the historical situation: the same rain fall event which now produces 5,000 liters/second would have produced only 3,000 liters/second some 20 years ago which she attributed largely to specific characteristics of the landscape (pavement, roof tops, roadways) with which she is very familiar [vii-viii]. She then returns to the summer situation and provides both situation description (“dry up” “farming cannot occur”) and a corresponding state in the sign domain (“below the first square”) [ix].

In this excerpt, we see Karen effortlessly moving back and forth from the chart, the sign domain, to the local setting, the referent domain. She reads the graph transparently, much like we read the newspaper: a simple glance at the material basis of the text suffices to instigate an interpretant in which elaborate situation descriptions tell us about the watershed. However, the process is even more complex than it first appears. Because the cross-section of the creek is not square, the water level curve does not directly indicate the amount of water in the creek. The relationship between the amount of water running past the station and the height of the curve is strongly non-linear. Therefore, when Karen reads the graph in terms of a specific quantity, she does this based in terms of a calibration curve established by a government agency. Thus, in her reading of the graph, not only the pen chart record, but also the calibration curve is transparent.

<u>“This peak has an earlier tiny peak. . .”</u>

The following excerpt is characteristic of the readings of scientists in that, to help the listener understand, the speaker provides a rich situation description which stands in relation to a particular feature of the graph (Figure 4.b). At this point, Karen takes part in an open house of an environmental activist group that has as its goal the preservation and improvement of the local watershed. She has brought four-years worth of graphs and posted one of the paper rolls from the pen chart recorder along the outer walls of the room. Her intent is to inform visitors about the work she is doing at the farm about improving the water quality and increasing water quantity available for farm use. As one visitor approaches her, Karen begins to read after a short introduction to situate the source of the graph.

> [x] This peak has an earlier tiny peak [8] and then the main one [9]. [xi] And from the data collection that I’ve done since I started that in 94, we know that the north arm has approximately 20% of the volume. It comes off Mount Nemo and then Grant [Creek] comes from the south has above 80% of the total volume up to the point where they meet. [xii] After they meet, from the beginning of the valley where you used to live, or

> you still live, to the end of the valley, the flow can increase 6 to 10 times all throughout the year. [xiii] That tells us that Mount Nemo and the First Nations forested area are extremely crucial for infiltration, ground water movement in feeding the creek throughout the year. [xiv] So, what this blip [8] means then is that when we get a rainfall, we had quite a bit there, the north arm, the water from the north arm comes off faster. [xv] It's less volume [8] but we see the peak first, down at Ocean Side Farm at the station. And then it takes quite a bit longer [9], 8 hours perhaps, and quite a lot to show that rainfall. [b]

Karen begins the episode [x] by making salient a tiny peak which precedes the main peak (Figure 4.b.). This in itself is significant because the non-initiate reader may not perceive the small peak, or may consider it as some random event. Whether such blips are to be read as signal (i.e., sign) or as noise (i.e., non-sign) depends on the situation at hand and is learned as part of local practice (Roth & McGinn, 1997, 1998). In Karen's reading, the blip is a non-random event and therefore a sign; here, the sign indicates certain geographical and physical characteristics of the creek, the sources of the water passing by her station (north arm, south arm, forested areas of Mount Nemo and Reservation), and water movement [xi-xiii]. She then returns to the "blip," which she reads (based on her 4-year experience in the valley) as the water coming from the north arm that precedes the water of the much longer south arm which also has to traverse a plain. She reads the relative size of the peaks as differences in the volume. Furthermore, the small size of the "blip" is implicitly attributed to the fact that the two tributaries only contribute one-sixth to one-tenth of the overall flow [xii].

"This is a non-natural event. . ."

Karen is not only familiar with the geographical area (about 18 km^2), but also with the measuring device itself and with farming practices that bring about more sudden changes in her graph (Figure 4.c). Thus, she attributes some graph features not to 'natural' events in the valley but also to sudden burst or stops of irrigation, technological problems in the monitoring station, the introduction and removal of a dam in the creek, and so on.

> [xvi] Here, this [10] is a non-natural event, a natural event has a duration [11], kind of has a roundness [11]. [xvii] So, what happened here [10] to the technical person, that's an obvious clogged pipe. [xviii] So the pipe goes across the stream, the water fills into the pipes and fills the stilling well, the stilling well drives the float, the float drives the pen on the paper. So it was clogging itself within an hour [12] a little bit. So that is what that means. . . [xix] Small rainfall events, ground is already saturated in the winter, we've had fall, winter rains, ground can't hold anymore. [xx] So, we see little, well surely substantial pieces [13] even though we have just a little tee weenie bit of rain, about a quarter of an inch [14].

Karen points out a peak (Figure 4.c) as a "non-natural event"; the very spikiness of the graph at this place, which contrasts the roundness of the curve in other places (as indicated in her gesture) is a sign to be read. Here, Karen's very language makes the peak a transparent window to the world: "this *is* a non-natural event" and "*that's* an obvious clogged pipe." Rather than seeing just spikes, graphical features provide Karen with direct access to states of the world (in the way we do not interpret a stop sign and then decide to stop, but rather stop). She elaborates this interpretant by providing a detailed description of the recording mechanism and how a malfunctioning station ("clogged pipe") translates in to the feature she had pointed out to her listeners [xvii-xviii]. When one of these visitors to the open house asked Karen a question:

V: Are these all, these are more natural events after, after POINTS[graph]?
K: Yeah, they're gradual curves, so like your rolling mountain type of thing, very. . . more natural events.

Standing in front of a section of the graph, the visitor asked whether "these are more natural events after" while pointing to a spike which Karen had identified three minutes earlier as a "not normal kind of thing" due to the opening of a dam. Karen ascertains that there are more natural events, and supports this with the description of the curves as of the "rolling mountain type." Here, Karen provided the visitor with a heuristic for recognizing natural events, though there are no indications anywhere else in our transcripts that her talk was causally determined by this or any other heuristic. It is further interesting that the visitor's question reflects the same kind of sign-referent transparency as Karen's discourse.

In the final part of the excerpt, Karen provides more evidence for her detailed understanding of the area's geology, its ability to retain water, and how high saturation levels during the winter months lead to higher peaks even for small rainfall events [xix-xx]. This excerpt makes evident again the detailed understanding of situation required to enact transparent reading. For even the assessment whether or not a peak is "substantial" relative to the amount of rain is not something that could be read of easily without the embodied knowledge that this water technician has developed as part of her work in the watershed.

In summary, Karen's reading is in many ways representative of the processes of reading graphs transparently which we recorded when scientists engaged with graphs from their own work. Whereas there are considerable philosophical problems when signs are conflated with things in the world (Bateson, 1980; Foucault, 1983), this same conflation usually gets the everyday work done in unproblematic ways. Each time, extensive situation-specific knowledge of the setting including tools and instruments used to collect data was used as resource to explain how their graphs should be read. Our own, two-year ethnography in which we followed an ecologist around observing all activities that ultimately led to conference presentations showed how much scientists use implicit understandings ("anecdotal knowledge") in their reading (Roth, 1998; Roth & Bowen, 1999b).

Graphs: Doing Puzzles and Explaining Work

In order to prevent misunderstandings, we insist that our video tapes recorded situated activities in both case studies: Karen and Ted engaged in reading of graphs displayed before their eyes. In both instances, the participants were in their familiar surroundings. Our video and transcripts are records of these activities. However, both sessions are further situated in a larger context. Here, the differences between the two readings become apparent. In Karen's case, there was a continuity with all the other things she normally does as part of her everyday practices. The relation of the variables in the sets {R, S, c, r} had those unique solutions that allow her to get her day's work done. On the other hand, Ted faced a different kind of task discontinuous from his everyday work and private life. He had to bootstrap a reading by structuring the graph and seeking suitable instances from his past experience such as to find plausible solutions to the relation $R = f_r(S,c)$. The discontinuity of between the sets {R, S, c, r} that characterize his everyday practices and those that he had to generate during the task, and the indeterminacy of the results of the bootstrapping process are plausible candidates that account for the problematic reading and the nature of the tasks to be "decontextualized."

Doing Puzzles

Students are frequently tested using mathematical tasks that involve descriptions of situations; these descriptions are said to make the tasks "contextual." When students fail to provide standard interpretation, deficit arguments are often rallied as explanations. Our research among scientists shows that if the graphs are unfamiliar, providing textual information does not itself guarantee successful standard interpretations and many errors occur which are similar to those committed by students. Given the nature of our participants, deficit arguments are certainly inappropriate. In an earlier study, we argued that "contextual" should refer to the extent to which the task provides resources that are part of a person's everyday activity (Roth, 1996). In order to better understand the sometimes crass differences we observed when scientists read our graphs versus their own, we unpack the situational differences in the two tasks as they relate to Ted and Karen.

We interviewed Ted in the familiar surroundings of his work, his office. Ted agreed to participate in our research and understands his interpretation as one that will help us better understand graphing and how to teach it (better) in school and university contexts. More than anyone else, Ted took the task as a challenge with the fervor of a puzzle aficionado. He began each of the task reading the Cartesian graph without attending to the caption which, in some instances, he covered up. Only when he felt he had said as much as he could did he read the caption. But it was clear that we, the investigators, were somehow in control of the standard interpretation. Our participants played the game of being interviewed for the benefit of future generations of students who might receive improved instruction in graphing.

In this situation, Ted (as our other participants) engaged in a task whose central elements are not from his everyday (work) life. Although he works with graphs on a regular basis, which provides him a familiarity with general graphical conventions, r, in the scientific community, Ted is neither familiar with the particular graphical relationships nor the standard referents of this graph within the ecology community. But because Ted wants to help us, he does what he can to produce a reasonable interpretation. In the process, he has to find possible values for R, S, c to satisfy the relation $R = f_r(S,c)$. Because different sets of values can satisfy the relation, Ted has to bootstrap his understanding using what is available as resources and constraints. From our case study we can see that he draws on everyday and mathematical resources to get the process started. For example, he generates "limits in the environment," "competition," "trapping," "fishing," and so forth as situations that have a relation to the particular shape of the graphical sign. He also generates “non-linear systems,” “chaotic behavior,” “family of curves” as interpretive resources from his work of modeling natural systems. We can then read our transcript of Ted’s activity as the protocol of an individual engaged in this bootstrapping process. This process involves entities that are assumed to be signs without knowing what they signify. This bootstrapping process is a tentative and, as we saw in Ted, sometimes hazardous (because of the arbitrary nature of signs) tracing of a system of signification rules by means of which sign-reference relations are established. Our protocol is therefore full of the mumbles and stumbles of real time activity in an unfamiliar environment. (See also Greeno [1991] for an environmental metaphor of situated knowing in a conceptual domain.)

In the present study, we used Karen to exemplify transparent reading of graphs. Her transparent reading makes it seem as if reading graphs from one’s own work was automatically transparent. We do not want to suggest this. For example, it is likely (although we do not have data to support this intuition) that Karen and her employer had to engage in a structuring process that allowed them to find a correspondence between some state in the world (here “clogged pipe”) and the sudden peak on the chart.

Explaining Work

When scientists interpreted their own graphs, the situation changed. They now moved on familiar terrain, knowing both the referent and sign domains. When the scientists and Karen talked about the graph from their work, they no longer simply provided uninterrupted readings. Although there were still considerable stretches when our participant read the graph, there were also moments where the other people present (interviewers, visitors to farm and open house) asked for elaborations and explanations. The two types of situation differed as to who controlled what was interesting relative to the issues at hand. We discuss the two situations separately.

When the scientists and Karen presented, explained, and elaborated the graphs, much of what they said concerned thick descriptions of the contexts to which their graphs referred and the phenomena the graphs stood for. We exemplified this situation in the case of Karen. Working in the community, knowing the farmers and their farming practices, as well as the physical, geographic, geological, and meteorological characteristics of the valley, provides Karen with a rich tapestry of experiential and embodied knowing of the situation. Even if unarticulated, this experiential and embodied knowing constitutes a background that is the source of “sense” and “meaning” attributed to such entities as signs, objects, events, and activities. As we found out on many excursions with her in and along the creek, to places overlooking the watershed, and observing her at work building riffles, she has an intimate knowledge of the watershed. Through her work, she is familiar with how particular events and conditions will change the creek (which she visibly inspects daily) and therefore the recordings of the pen chart. Each aspect of the graph (sign) has its place in a network of significant events and practices, characteristics of the recording device, how water level translates into the recording and its relation to the total rainfall. Therefore, when Karen reads the graph for the visitors to the farm at the open house of environmental activists, we witness someone operating in her everyday context. Here, signs function against a background that the signs presuppose. In such situations, signs cannot be understood as mere relations; rather, signs and their signification are an integral part of the context and they can be a sign only for those who dwell in this context (e.g., Dreyfus, 1991).

Karen's reading of the chart thereby has become transparent such that she is no longer concerned with the material details of the signs themselves, but her reading leaps beyond to the world she knows so well. The reverse movement from the world she knows to the expressive domain also continuously happens. Provided any situation description, Karen projects without hesitation what the graph will look like. Although she has not lived in the area for 20 years, Karen knows from the older residents that the valley has experienced a housing and road-building boom. Without deliberating, she knows what the graph would have looked like. Furthermore, elsewhere in our database, she discusses the impact of a projected housing development which would increase the number of inhabitants by 5,000: the impact would be disastrous as the water flow patterns would increase even above the currently observed peaks, and the run-off would be even faster. These events would then characterized by higher and narrower graphical peaks. Even "non-natural events" are transparently read from the chart [xvi]. Although she may not have seen such a graphical feature before, Karen will actually have a reasonable answer if she is questioned about a line that does not stand in a reciprocal relation with a natural event. This was the case in the following exchange with a visitor. (Her interlocutor at this point is Walter, an inhabitant of the valley and mayor of the community.)

K: You get a bit of rain, you're gonna take advantage of that rather than extract. . .
W: Yeah and that's still in June, so that's still not be, there'd still be water in there, so. What's the blue line, just an ink thing?
K: Yeah, yeah. Good question. I've never seen that before, my eyes peeling for the black line. It's just the end of the roll 'cause I've got it taped right here.

Here, Karen was so much in her world, the one which includes the (black) curve, the water flow, and the watershed which produces it, that she was oblivious to the blue streak made salient by Walter. Though she had not attended to the blue line, standing in front of a joint where two rolls pen chart were taped together, Karen attributed the streak to the end of the roll without detailing why this might be so. From her experience, such a blue streak cannot have a referent in the watershed so it disappeared from her lifeworld. Its referent has to be in the recording mechanism and is therefore an artefact. Karen is not only familiar with the graphs as entities that signify, but especially with the particulars of the setting that led to the graphs. Through her work, many if not most of the practically possible sets of values satisfying the relation $R = f_r(S,c)$ are so familiar that she no longer distinguishes between signs and what they stand for. Her activity, as those of all scientists when they read graphs from their own work, was situated in the sense that they were dealing with familiar entities: the relevant signs, the objects and events these signs referred to, and the conventions regulating sign use.

When someone in the respective "audience" decided to ask a question, the control over salient issues changed. Although the scientists and Karen were still in control of the graphical display (qua experts), the settings to which the graph pointed were no longer under their control. What was relevant at the moment, and how the information from the graph related to the world more broadly, very much emerged from the interaction between the presenting person and the audience. In the following transcript, Karen had begun to point to a step in the curve explaining that on this day, the people on her farm began to irrigate the field.

K: June 7th, this is when we began irrigating, full irrigation is starting here and so=
W: =How many places in the creek are you pulling it out?
K: Just, we only monitor the quantity taken out at Ocean Side Farm, we have no idea what's going on upstream. So we have a water meter that shows how much we're extracting and we keep track, we have the pump on=
W: =That would be interesting in finding out, trying to get a rough idea of what other people are pulling out. There is not much you can, you might not be able to quantify it but at least just knowing. The dates that people pull out certain amounts, the multiple pumps that are working and the water they push out in an hour.
K: We kind of try to do that, like BC Environment has a list of all the licensed service. . . licenses and people with irrigation ponds and storage ponds and. . . We kind of did a number and if anyone, if everyone were actually pumping those amounts at the same time.
W: I used to have a license.

Although Karen wanted to go on and finish explaining the (to her) salient parts of the entire chart, Walter's questions (as those of other people present) codetermined what was interesting and being talked about. Thus, Walter was interested in finding out about the number of places where water is extracted from the creek and which therefore gives rise to the signal before them. Karen did not have an answer to this question but, in turn, discloses that her farm keeps track of the amount of water drawn from the creek. But Walter insisted on stating that it would be interesting to know the exact number of users, the date and time of use, and the amount of water taken by each user.

While such questions and elaborations did not elucidate the graphs qua representations, they did a lot to elaborate the referent domain. They have become—in the words of Heidegger (1977), a philosopher who was, more than anyone else, concerned with everyday activity—pieces of equipment that are ready-to-hand and which scientists therefore use transparently. As equipment, the signs point out the context of practical activity which the scientists share with others. That is, it was here that there was considerable intersection between the scientific expert of the graph and the persons in the audience. It was at this point that general interests in ecological questions surfaced and were discussed rather than questions of S-R mapping or the nature of S. It is from these interactions that we actually found out a lot about the understandings our participants had developed in the respective referent domains.

Discussion

In this study, we take a semiotic perspective on reading graphs which is grounded in the literature on reading texts and graphics, that is, semiotics and hermeneutic phenomenology. We therefore conceptualize graphs and their captions (and other accompanying text) as multimodal texts which are read by individuals of varying background. Each reader engages in semiotic and hermeneutic processes. In this way, we recover the individual mathematical experience all the while keeping the sociocultural dimensions of knowing in focus. Within this semiotic model, we expect trouble when the reader (a) is unfamiliar with the sociocultural conventions r regulating sign use; (b) is unfamiliar with real or possible situations, R, signs refer to; and (c) does not produce interpretants I' (and their context c) such that they become signs S relevant to the standard reading.

Using Signs

Consistent with conventions in semiotics and philosophy, sense and meaning are not synonyms. Sense is given by the relationship of a sign to its interpretant(s), that is, the place a sign takes with respect to other signs, and therefore its place in language games (Wittgenstein, 1994). Meaning includes both sense (S-I) and reference (S-R) dimensions of a sign and requires a sufficiently large rhizomatic or mazelike network of sense and reference relations (Eco, 1984; Nöth, 1990). Ordinarily, when we read in everyday situations, a newspaper text for instance, words and numbers are transparent. That is, we read, but leap beyond the text itself to the things that the text speaks about. For example, when literate English readers read "Seven years ago, the average debt carried by a post-secondary student upon graduation was $8,000. Today it's $25,000" (Danard, 1998, p. A3), they know what the text is about without worrying about the words themselves or the type face. They instantly know that the difference between $8,000 and $25,000 is "seventeen thousand dollars" (Unless, of course, one is from a continental European country and unfamiliar with the reverse use of "," and "." to mark decimal position and sequences of "000"s), that it takes a considerable time to pay off such a loan, that one has to have a decent job to be able to carry the loan, etc. They also know that if you study at a university with a trimester system, this amount may be smaller, and if you take longer or continue on to complete a graduate degree, this amount will be larger.

In this example, the material basis and shape of the signs that make the newspaper text are transparent and our reading leaps beyond to the world of university education that the text is known to be about. In our research, scientists' readings of their own graphs (exemplified in the case of Karen) was most similar to the newspaper example. Here they moved freely between expressive and referent domains. For Karen, the valley as the familiar lifeworld and her everyday practices of working with the pen chart record have become fused. Such fusion appears to be related to extensive, situated experiences in both domains and with the translations that provide the mapping (Nemirovsky et al., 1998; Roth, 1996). On the other hand, the case study of Ted shows that even scientists do not read unfamiliar graphs transparently even if they were designed for undergraduate education in their own domain. Here, it is not only that the particular graph and the elements from which it is constituted (expressive domain) but also the referent domain that was unfamiliar. Despite extensive experience in plotting data related to intimately familiar natural phenomena, Ted's reading of the unfamiliar graph involved considerable efforts which were not always successful within a normative frame.

We showed how Ted's reading moved back and forth between the structuring process (identifying aspects that signify) and the process of grounding signs in the world. Both processes are characterized by dialectic stabilizing movements. An interpretation emerges as the reader seeks a translation from the expressive domain, and by finding appropriate contextual constraints that decrease the number of solutions in the relation of R, S, c, and r. In the process, we saw that what is a sign is not always certain nor whether a sign required by the understanding of the situation is actually available in the graphical representation. We also observed how familiar practices allowed Ted to read some aspects of the graph in a transparent way such that it became difficult to assess whether the utterances "birth rate" or "death rate" referred to some state in the world or to the graphs at hand. The case of Karen clearly showed how reading a graph competently is bound up with her work practices and her personal experience in the watershed.

"Misreading" Signs

Previous research showed how students, for a variety of reasons, "misread" graphs. One source of problems arises when the relevant signs are not perceived, another when the context constituted by other signs is not individuated, so that the relevant constraints supporting a canonical reading do not exist. However, we have seen that Ted also "misread" the population graph (as did a number of his colleagues). Thus, he initially read "$b - d < 0$" (Section III, Figure 3) and inferred from it a population collapse rather than a dynamic situation in which N, b[N], and d[N] are continuously adjusted until an equilibrium state is reached and the population is maintained at or near the second intersection point at which $b = d$. Ted recovered from this inference only after constructing his own graph that made his readings of the different sections and intersections incompatible with each other.

Another form of "misreading" identified among students are slope/height errors (e.g., Leinhardt et al., 1990). As previously indicated, this error comes about when students are asked to read something from a graph that is designed to exhibit another relationship. Thus, the information sought for is only available implicitly and, in the case of velocity from position time graphs, only because of the co-incidence of derivative and the definition of velocity. From a perspective of graphs as rhetorical pieces (e.g., Latour, 1987) and as our respondents generally pointed out, using one graph when the core issue can be made more salient in another type of graph is "bad practice." Thus, scientists committed the equivalent error to slope/height confusion when asked where the maximum number of individuals would be added to an existing population. Ted and all but one of his colleagues pointed to b_{max} or $(b-d)_{max}$ rather than to the appropriate $[(b-d)\cdot N]_{max}$. Here, scientists similar to school children, used a salient feature of the graph (an existing maximum) in answer to a question that more easily could be read off another type of graph. In fact, slope/height errors may be classified as iconic. It may turn out that the question "Which is more. . .?" biases (in the sense of favors) the participants' attention and perception processes to a graphical representation of "more" in the same way that a representation of, for example, a curve on race track biases attention and perception to curves in a Cartesian velocity-time graph.

a. b.

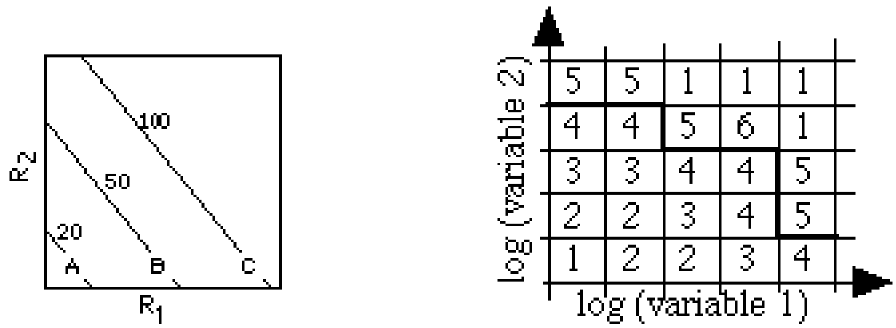

Figure 5. a. An isograph task showing the interaction of two resources (R_1, R_2) on a third variable (growth rate). **b.** A table representing the effects of two variables on a third as it appeared in a scientists publication. One isocline is drawn in boldface.

Because competence and transparence of reading graphs was related not only to familiarity of graphs but also with the understanding of the domain to which they refer, we do not expect that students (or any individual) easily develop ways to enact practices appropriated in one domain to another domain. In one instance, an experienced public-service scientist who had received an international award for the publication from which he read graphs, did not perceive the structural identity between one of his representations and that of one of our tasks (Figure 5). Both our isocline graph (Figure 5.a) and the scientist’s table (Figure 5.b.) represent the interaction of two variables on a third. The difference is that in his representation, continuous variables had been collapsed so that each category comprises a range of the continuum. Thus, what was an isomorphism in our own perception, was something different and unrelated from that scientist’s perspective. Transfer had not occurred. Changing the referent domains while holding constant the structure of the expressive domain is not easy. Even experienced professors have been shown to elaborate situation descriptions intended as analogies, but which did not have the same structure as the original scientific phenomenon (Roth & Bowen, 1999a; Roth, Tobin, & Shaw, 1997). In the process of elaborating a new situation, and unbeknownst to the professors, the structure of the new domain no longer corresponded to the required structure given the graphs and the S—>R translations to be explained by the lectures.

Based on these research results and our theoretical framing, we do not expect students inexperienced in the conventional practices of graphing, nor in the possible nature of signs or the phenomena the signs are about, to infer the structure of the translations and the unfamiliar content domain. This is because all representation is at the same time interpretation and translation. If we do not recognize it as such, this is because of our familiarity with the representations that have become transparent in our activities (i.e., Karen would no longer consider her reading of a squiggle as the opening of a dam as translation or interpretation). But it is important to keep in mind that even the apparent visual similitude between the outline drawing of a horse and some living horse is produced, consistent with some cultural decision, and therefore has to be learned (Eco, 1976). Thus, geometric similitude and topological isomorphism both are transformations which require rules and conventions. These rules and conventions specify how a point in effective space of the expression is translated and therefore corresponds to a point in the virtual space of the content model; and they require considerable familiarity with the network of interpretants required for meaning to exist. Take the three examples in Figure 6. National debt and GNP are frequently expressed on the same axis and in the same direction (Figure 6.a) drawing on conventions similar to those that underlie the birth rate and death rate curves (Figure 6.b) used in the present research. On the other hand, physicists and chemists express the two forces that are responsible for the Leonard Jones potential, repulsion and attraction as curves with opposite signs (Figure 6.c). To find the net population rate, for example, death rate has to be *subtracted* from birth rate for a particular value of the population density N_1 (i.e., $b[N_1] - d[N_1]$); to find the net force for a particular radius R_1, the values of the two forces have to be added (i.e., $F_r[N_1] + F_a[R_1]$).

Transformations may differ by the mode of correspondence and the class of elements made salient (and therefore pertinent) by the conventionalizing procedure; some elements are retained as invariant whereas others are varied. Some transformations aim to preserve geometric properties, others metric properties, still others topological properties, and so on. It must be retained that transformations are not natural correspondences, but require knowledge of the rules and conventions by which an expression relates to some content. They also require that the reader attends to and perceives those features in both domains that allow such a translation to occur. Neurophysiological research shows that both attention and perception are functions of prior knowledge and experience (Jarvilehto, 1998). Thus, without knowledge and experience in both graph and referent domains, we expect interpretive processes to be stymied. Through interpretation, a reader's understanding does not change into something else or reveal new information. Rather, through the interpretative process, the reader works out possibilities already projected in the understanding brought to the graphing task.

Previous research shows that when students are familiar with mapping nature in the form of graphs, and have extensive knowledge of a domain of natural phenomena, they may be much more competent than B.Sc. graduates in the interpretation of data and graphs (Roth, 1996). Once

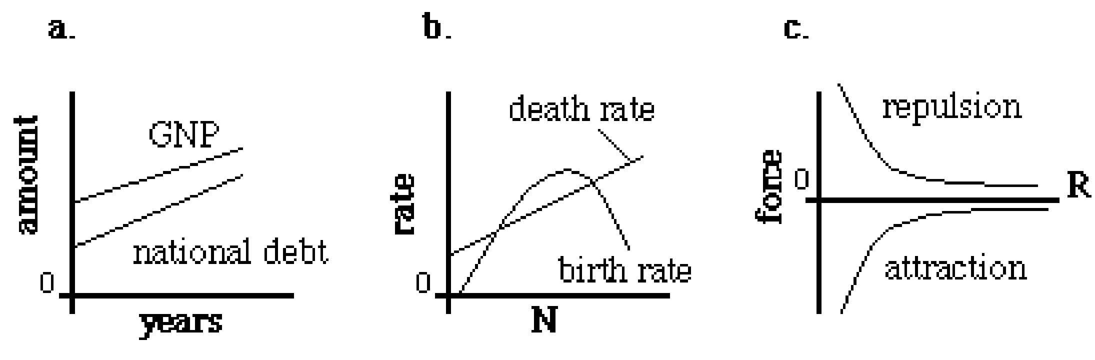

Figure 6. Different communities, different rules and conventions. The dichotomous variable pairs GNP-national debt and birth rate-death rate are expressed in terms of the same sign (+) whereas the equivalent repulsion-attraction are expressed in terms of opposite signs (+/-).

representations are overlearned we no longer recognize their constructed and learned nature. Thus, Karen could point to a feature of the graph and state, “This *is* a non-natural event” and then provide a detailed situation description of the measurement device and its functioning. We engage in this process everyday in the context of natural language use. We no longer note that we treat maps (language, graphs, numbers) as if they were the territory (Bateson, 1980). However, that this transparency is precarious was a central thesis of Rene Margueritte’s paintings which feature a pipe and underneath it the text “Ceci n’est pas une pipe” [This is not a pipe] (cf. Foucault, 1983). First, the representation (drawing) of a pipe is not a pipe. Second, the sentence is valid even if mounted below a physical instantiation of a pipe, because the class “pipe” contains more than one member. Contrary to mathematics and other formal languages, natural language does not make clear distinctions between the name of a class and the name of any one of its members.

Implications

Where does this leave the mathematics (science, geography) teacher who wants students to develop competent graphing practices? Our research shows that competent readings are related to an understanding of the phenomena signified as well as the structure of the signifying domain, familiarity with the conventions relating the two domains, and familiarity with translating between the two domains. Graphs as a whole and local features of them are not significant (signifying!) signs on their own. Rather, what is a signifying aspect of a graph depends on the targeted referent domain, in part available through the verbal texts (e.g., labels, units, variable names, captions) that accompany the graphs. Furthermore, whether a blip on a graph is a sign or simply a spurious event involves deep familiarity and embodied knowledge of the domain, the data collection devices, representation-producing mechanisms, expressive domain, feature-enhancing techniques, and translations between them (Roth & McGinn, 1997, 1998). On the other hand, there is reason to believe that if students do not have these experiences, they are likely to remain referentially stuck within the expressive domain, never developing the competence to project to other expressive domains or to the phenomenal world (Greeno, 1989; Kaput, 1988). Finally, it is only through the continuous movement between the experiential and expressive domains that we expect students to begin to dissociate the features of the two which lead, without familiarity in translating, to iconic errors.

All of this appears to require much more than traditional instruction in graphing has allowed for. To read a graph competently, it is not sufficient to have received some instruction on the mechanical aspects of producing graphs; extensive interaction with phenomena and representational means seem to be prerequisite for competent graphing practices. In support of this contention, there are few examples in real classrooms where students were provided with extensive experiences allowing them to translate between the expressive and referent domains (e.g., Hall & Stevens, 1995; Mokros & Tinker, 1987; Roth, 1996). In these cases (as in the few laboratory studies [e.g., Nemirovsky et al., 1998]) students have been shown to concurrently develop a deep understanding of the referent domain (how the world including the measurement devices operate), the expressive domain (graphs and different ways of constructing features as salient), and translations between the two. Further research is required to understand the relationship between mathematical representations and experiences in the world, and how this relationship changes with increasing familiarity in both domains.

Throughout this paper, we have emphasized that competent graphing practices require familiarity with signs and their referents. However, we do not intend to express that graphing in particular and mathematics more generally has to have referents in the natural world—though as natural scientists and applied mathematicians, we see little wrong with such intentions. Our central point to be considered for curriculum design in mathematics is that signs are used to communicate and, in this, are embedded in the shared practical activities of people. Some of these people are (like) mathematicians whose activity is concerned with the properties of signs and sign systems. Others are like ecologists concerned with things of the natural world. In both cases, the activities ground the mathematical signs and make them meaningful are deeply embedded in the social and historical contexts of the discipline. The goal of the mathematics curriculum should therefore be the creation of opportunities such that students can participate in and establish communities in which signs are used to constitute patterns in mathematical and natural worlds, and for managing the required social relations and communicative activity. We therefore need to focus on participation in consensual mathematical activity made available through actions of doing intelligible mathematics rather than in individualistic appropriation (construction, acquisition, etc.) of static signs and their grammar.

Acknowledgments

This work was made possible in part by Grant 410-96-0681 from the Social Sciences and Humanities Research Council of Canada. Our thanks go to the scientists who agreed to volunteer in this research project and to Sylvie Boutonné who recorded and transcribed most of the videotapes.

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